Exam Quantum Physics II

Thursday, November 2, 2006, 9:00-12:00.

Before you start, read the following:

- There are 4 problems with a total of 50 points.
- Write your name and student number on every sheet of paper.
- Write the solution of each problem on a separate sheet of paper.
- Illegible writing will be graded as incorrect.
- *Good luck!*

Problem 1 (*45 minutes; 15 points in total*)

Answer the following questions, brief and to the point:

2 pnts (*a*) Prove that $[J^2, S_z] = 2i\hbar(\vec{S} \times \vec{L})_z$ where $\vec{J} = \vec{L} + \vec{S}$.

2 pnts (*b*) Give the possible wave functions of two free electrons, taking into account the Pauli principle.

2 pnts (*c*) Write down the Hamiltonian of the helium atom. What is the ground-state energy, in formula and in eV, when the interaction between the electrons is neglected?

2 pnts (*d*) Formulate the spin-statistics theorem. Give two examples of a boson, and three examples of a fermion.

2 pnts (*e*) Describe the principle of an NMR experiment and how it can measure the g-factor of the proton.

2 pnts (*f*) A carbon atom has two p electrons in the outer shell. Which of the possible terms ${}^{2S+1}L_J$ are allowed by the exclusion principle?

2 pnts (*g*) Consider (time-independent, nondegenerate) perturbation theory for a Hamiltonian of the form $H = H_0 + \lambda H'$. Give the formula for the first-order correction to the energy E_0, and explain in words what it says.

1 pnt (*h*) Where is Schrödinger's cat?

Problem 2 (*45 minutes; 15 points in total*)

The Lyman series in hydrogen is the series of spectral lines that correspond to transitions $n' \to n$ to the ground state $n = 1$.

4 pnts (*a*) Calculate the energy in eV, and the wavelength in nm, of the Lyman-α line ($n' = 2 \to n = 1$) and of the limit of the series $n' \to \infty$. Use $\alpha = 1/137$ and $\hbar c = 200$ eV·nm. In which part of the electromagnetic spectrum do these lines lie?

2 pnts (*b*) Calculate the relative difference of the wavelengths of the Lyman-α line for deuterium and for hydrogen.

Consider next the fine-structure of the hydrogen spectrum. The energies are given by

$$E_{n\ell j} = -|E_n| \left[1 + \left(\frac{Z\alpha}{n} \right)^2 \left(\frac{n}{j + \frac{1}{2}} - \frac{3}{4} \right) \right],$$

where E_n are the Bohr energies, $Z = 1$, and $j = \ell \pm 1/2$.

3 pnts (*c*) Discuss (no derivations!) which two physical effects are responsible for the fine-structure.

3 pnts (*d*) Calculate the fine-splitting of the $n = 1$ and $n = 2$ Bohr levels by giving the shifts with respect to the corresponding Bohr energies in cm^{-1}. Use that 1 Rydberg corresponds to 1.1×10^5 cm^{-1}.

3 pnts (*e*) Give the dipole selection rules for fine-structure levels (no derivation!). Out of how many, and which, lines does the Lyman-α line consist? Make a schematic drawing of the levels involved and indicate the transitions.

Problem 3 (*35 minutes; 10 points in total*)

An electron is at rest at the origin in an eigenstate α_x of S_x, with eigenvalue $+\hbar/2$. At time $t = 0$ it is placed in a magnetic field pointing in the z-direction, $\vec{B} = (0, 0, B)$, in which it is allowed to precess for a time T.

2 pnts (*a*) Give the time-dependent Schrödinger equation for the spin vector $\xi(t)$. Write the Hamiltonian $H = -\vec{\mu} \cdot \vec{B}$ explicitly as a 2×2 matrix.

4 pnts (*b*) Solve the equation, taking into account the appropriate boundary condition at $t = 0$. Use the notation of the cyclotron frequency: $\Omega = |e|B/mc$. What is $\xi(T)$?

At this time $t = T$ the magnetic field is very rapidly rotated in the y-direction, so that now its components are $(0, B, 0)$. After another time interval T a measurement of S_x is carried out.

4 pnts (*c*) Give in Dirac notation the probability that the value $+\hbar/2$ will be found, and calculate it.

Problem 4 (*25 minutes; 10 points in total*)

5 pnts (a) Write during 10 minutes (max. about 150 words) about the question: *What is a photon?*

5 pnts (b) Write during 10 minutes (max. about 150 words) about the question: *What is spin?*